Refer to NCETM Singapore bar article

Refer to NCETM Progression in Reasoning document

Addition

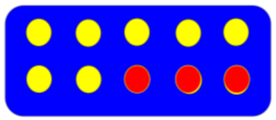
Year 1

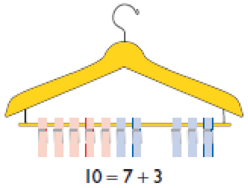
Mental Strategies (addition and subtraction)

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.



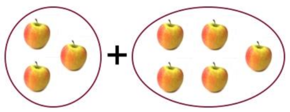


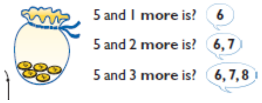
Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.

Children have opportunities to explore partitioning numbers in different ways.

e.g. 7 = 6 + 1, 7 = 5 + 2, 7 = 4 + 3 =

Children should begin to understand addition as combining groups and counting on.





Vocabulary

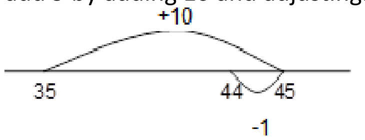
Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.

Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 7 + 3 = 10 to find 17 + 3= 20, 70 + 30 = 100

They should use concrete objects such as bead strings and number lines to explore missing numbers – 45 + __ = 50.

As well as number lines, 100 squares could be used to explore patterns in calculations such as 74 +11, 77 + 9 encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.

Children should learn to check their calculations, by using the inverse.

They should continue to see addition as both combining groups and counting on.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, Tens, ones, partition Near multiple of 10, tens boundary, More than, one more, two more… ten more… one hundred more

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.

Children should continue to partition numbers in different ways.

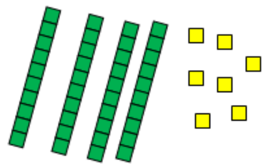
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

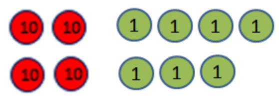
Add the nearest multiple of 10, then adjust such as 63 + 29 is the same as 63 + 30 – 1;

counting on by partitioning the second number only such as 72 + 31 = 72 + 30 + 1 = 102 + 1 = 103

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg.

What's the same? What's different?





Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)
When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

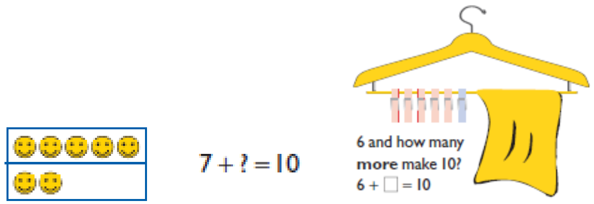
Another example here…

Some Key Questions

How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…?
How much more is…? One more, two more, ten more…
What can you see here?
Is this true or false?
What is the same? What is different?

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



Some Key Questions

How many altogether? How many more to make…? How many more is… than…? How much more is…?
Is this true or false?
If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).
What do you notice? What patterns can you see?

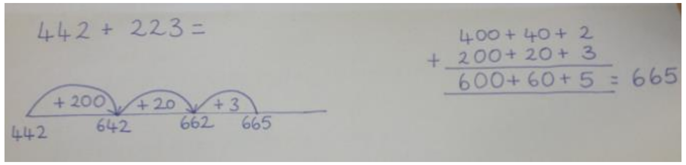
Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
Odd + odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?



Refer to NCETM Progression in Reasoning document

Addition

Year 4	Year 5	Year 6
Mental Strategies Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways. They should be encouraged to choose from a range of strategies: • Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77 • Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3) • Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7 • Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70 • Partitioning: compensating – 138 + 69, 138 + 70 - 1 • Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10 • Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm? • Using known facts and place value to find related facts. **Vocabulary** add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.	**Mental Strategies** Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways. They should be encouraged to choose from a range of strategies: • Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55 • Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6 • Partitioning: counting on or back - 540 + 280, 540 + 200 + 80 • Partitioning: bridging through multiples of 10: • Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1 • Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1 • Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20? • Using known facts and place value to find related facts. **Vocabulary** tens of thousands boundary, Also see previous years **Generalisation** Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate a – b = (a-1) – (b-1) represented visually. **Some Key Questions** What do you notice? What's the same? What's different?	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 **Vocabulary** See previous years **Generalisations** Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

Generalisations Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Can you convince me? How do you know?	

Refer to NCETM Singapore bar article

Refer to NCETM Progression in Reasoning document

Subtraction

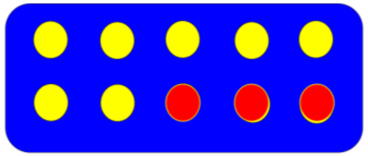
Year 1

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

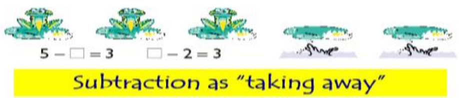
Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

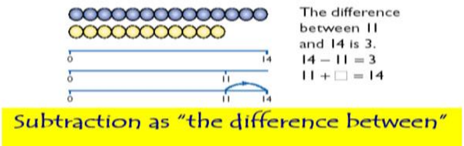
They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.





Vocabulary

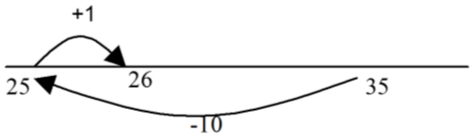
Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.



Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 10 - 7 = 3 and 7 = 10 - 3 to calculate 100 - 70 = 30 and 70 = 100 - 30.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

As well as number lines, 100 squares could be used to model calculations such as 74 – 11, 77 – 9 or 36 – 14, where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that ‘up’ equates to ‘add’.

Children should learn to check their calculations, including by adding to check.

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in difference ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201 – 198; counting back (taking away / partition into tens and ones) for 201 – 12.

Calculators can usefully be introduced to encourage fluency by using them for games such as ‘Zap’ [e.g. Enter the number 567. Can you ‘zap’ the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd – odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

What do you notice? What patterns can you see?

Generalisations

- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

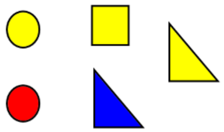
Children could see the image below and consider, “What can you see here?” e.g.

3 yellow, 1 red, 1 blue. 3 + 1 + 1 = 5

2 circles, 2 triangles, 1 square. 2 + 2 + 1 = 5

I see 2 shapes with curved lines and 3 with straight lines. 5 = 2 + 3

5 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 3



Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…?

What can you see here?

Is this true or false?

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus

Tens, ones, partition

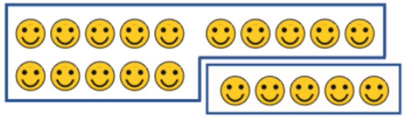
Near multiple of 10, tens boundary

Less than, one less, two less… ten less… one hundred less

More, one more, two more... ten more... one hundred more

Generalisation

- Noticing what happens when you count down in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$15 + 5 = 20$

Some Key Questions

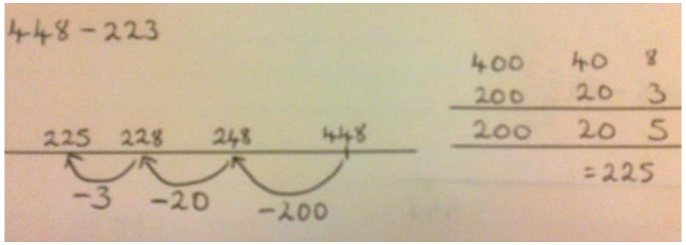
How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many fewer is… than…? How much less is…?

Is this true or false?

If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc).

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What’s the same? What’s different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line



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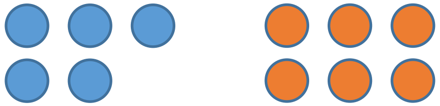
Subtraction

Year 4	Year 5	Year 6
Mental Strategies Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways. They should be encouraged to choose from a range of strategies: • Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77 • Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3) • Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7 • Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70 • Partitioning: compensating – 138 + 69, 138 + 70 - 1 • Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10 • Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm? • Using known facts and place value to find related facts. **Vocabulary** add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.	**Mental Strategies** Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways. They should be encouraged to choose from a range of strategies: • Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55 • Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6 • Partitioning: counting on or back - 540 + 280, 540 + 200 + 80 • Partitioning: bridging through multiples of 10: • Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1 • Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1 • Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20? • Using known facts and place value to find related facts. **Vocabulary** tens of thousands boundary, Also see previous years **Generalisation** Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate a – b = (a-1) – (b-1) represented visually. **Some Key Questions** What do you notice? What's the same? What's different?	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 **Vocabulary** See previous years **Generalisations** Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

Generalisations Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Can you convince me? How do you know?	

Refer to NCETM Progression in Reasoning document

Multiplication

Year 1	Year 2	Year 3
Mental Strategies Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with numbers in 2, 5 and 10 times tables They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.  Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower) **Vocabulary** Ones, groups, lots of, doubling repeated addition groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide …etc) **Generalisations** Understand 6 counters can be arranged as 3+3 or 2+2+2 Understand that when counting in twos, the numbers are always even. **Some Key Questions** Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Mental Strategies** Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Number lines should continue to be an important image to support thinking, for example Children should practise times table facts 2 x 1 = 2 x 2 = 2 x 3 = Use a clock face to support understanding of counting in 5s. Use money to support counting in 2s, 5s, 10s, 20s, 50s **Vocabulary** multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows **Generalisation** Commutative law shown on array (video) Repeated addition can be shown mentally on a number line Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Mental Strategies** Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged. Children should practise times table facts 3 x 1 = 3 x 2 = 3 x 3 = **Vocabulary** partition grid method inverse **Generalisations** Connecting x2, x4 and x8 through multiplication facts Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this. When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

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Multiplication

Year 4	Year 5	Year 6
Mental Strategies Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. Become fluent and confident to recall all tables to x 12 Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?) Use of finger strategy for 9 times table. Multiply 3 numbers together The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication **Vocabulary** Factor **Generalisations** Children given the opportunity to investigate numbers multiplied by 1 and 0. When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Mental Strategies** Children should continue to count regularly, on and back, now including steps of powers of 10. Multiply by 10, 100, 1000, including decimals (Moving Digits ITP) The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies to solve problems mentally: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table) **Vocabulary** cube numbers prime numbers square numbers common factors prime number, prime factors composite numbers **Generalisation** Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 They should be encouraged to choose from a range of strategies to solve problems mentally: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table) **Vocabulary** See previous years common factor **Generalisations** Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Understanding the use of multiplication to support conversions between units of measurement. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

[Refer to NCETM Progression in Reasoning document]

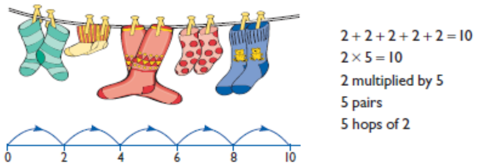
Division

Year 1

Mental Strategies

Children should experience [regular counting] on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



Children should begin to understand division as both sharing and grouping.

Sharing – 6 sweets are shared between 2 people. How many do they have each?

Grouping-
How many 2's are in 6?

They should use objects to group and share amounts to develop understanding of division in a practical sense. E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?

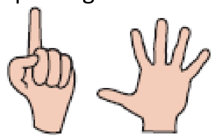
Children should begin to explore finding simple fractions of objects, numbers and quantities.

E.g.*16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?*

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.
Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:



> Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping.

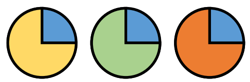
15 pencils shared between 3 pots, how many in each pot?

Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

[Use children's intuition to support understanding of fractions as an answer to a sharing problem.]

3 apples shared between 4 people = $\frac{3}{4}$



Year 3

Mental Strategies

Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables.
This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).

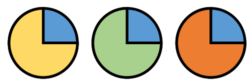
Children will make use multiplication and division facts they know to make links with other facts.
3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3
30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30

They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to given as a whole number) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

[Use children's intuition to support understanding of fractions as an answer to a sharing problem.]

3 apples shared between 4 people = $\frac{3}{4}$



Vocabulary

See Y1 and Y2
inverse

Generalisations

Inverses and related facts – develop fluency in finding related multiplication and division facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array **Generalisations** • True or false? I can only halve even numbers. • Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing. **Some Key Questions** How many groups of…? How many in each group? Share… equally into… What can do you notice?	**Vocabulary** group in pairs, 3s … 10s etc equal groups of divide, ÷, divided by, divided into, remainder **Generalisations** Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables) An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?) Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group. **Some Key Questions** How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number? If 12 x 2 = 24, what is 24 ÷ 2? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)	**Some Key Questions** Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?) What is the missing number? 17 = 5 x 3 + __ __ = 2 x 8 + 1

Refer to NCETM Singapore bar article

Refer to NCETM Progression in Reasoning document

Division

Year 4

Mental Strategies

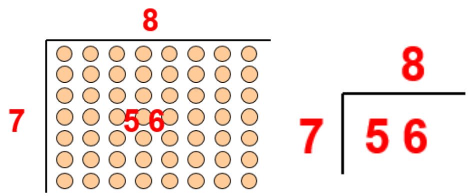
Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.
Children should learn the multiplication facts to 12 x 12.

Vocabulary

see years 1-3
divide, divided by, divisible by, divided into
share between, groups of
factor, factor pair, multiple
times as (big, long, wide …etc)
equals, remainder, quotient, divisor
inverse

Towards a formal written method

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.



Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.

Each digit as a multiple of the divisor

'How many groups of 3 are there in the hundreds column?'
'How many groups of 3 are there in the tens column?'
'How many groups of 3 are there in the units/ones column?'

Year 5

Mental Strategies

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.
Children should practice and apply the multiplication facts to 12 x 12.

Vocabulary

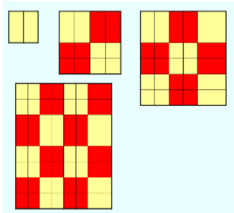
see year 4
common factors
prime number, prime factors
composite numbers
short division
square number
cube number
inverse
power of

Generalisations

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.
Start: **24 = 24**
Player 1: **4 x 6 = 24**
Player 2: **4 x 6 = 12 x 2**
Player 1: **48 ÷ 2 = 12 x 2**

Sometimes, always, never true questions about multiples and divisibility. E.g.:

- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.
- When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)



Year 6

Mental Strategies

Children should count regularly, building on previous work in previous years.
Children should practice and apply the multiplication facts to 12 x 12.

Vocabulary

see years 4 and 5

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

Some Key Questions for Year 4 to 6

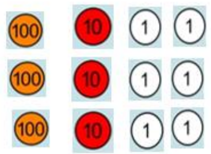
What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

$$\begin{array}{r|l} & 1\ 1\ 2 \\ \hline 3 & 3\ 3\ 6 \end{array}$$



When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto ‘carrying’ their remainder across to the next digit.

<u>Generalisations</u>

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

Inverses and deriving facts. ‘Know one, get lots free!’ e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

Sometimes, always, never true questions about multiples and divisibility. <u>(When looking at the examples on this page, remember that they **may not** be ‘always true’!)</u> E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.